# Unit 9: Algebra

## Lesson 1: Finding a rule (1)

➜ pages 64–66

**1.** a)

| Number of cakes | 1 | 2 | 3 | 5 | 10 | 100 | 1,500 |
|---|---|---|---|---|---|---|---|
| Number of stars | 1 × 3 = 3 | 2 × 3 = 6 | 3 × 3 = 9 | 5 × 3 = 15 | 10 × 3 = 30 | 100 × 3 = 300 | 1,500 × 3 = 4,500 |

b) For *n* fairy cakes, you need *n* × 3 stars.

**2.**

| Number of cakes | 5 | 6 | 12 | 20 | 101 | *b* |
|---|---|---|---|---|---|---|
| Number of stars | 25 | 30 | 60 | 100 | 505 | *b* × 5 |

Children should draw a picture of fairy cake with 5 stars on it.

**3.** Patterns matched to rules:
Top pattern ➜ *n* × 4
2nd pattern ➜ double *n*
3rd pattern ➜ 3 × *n*
Bottom pattern ➜ *n* × 5

**4.**

| Minutes Zac has been painting | 45 | 50 | 90 | 120 | *x* |
|---|---|---|---|---|---|
| Minutes Kate has been painting | 15 | 20 | 60 | 90 | *x* – 30 |

If Zac has been painting for *x* minutes, Kate has been painting for *x* – 30 minutes.
If Kate has been painting for *y* minutes, Zac has been painting for *y* + 30 minutes.

**5.** a) *b* × 8
*x* × 3
*m* × 7
*k* × 52

b) The number of days in *d* years is 365 × *d*.

**6.**

| | | | | |
|---|---|---|---|---|
| 1 | 3 | 12 | 15·5 | *x* |
| 5 | 7 | 16 | 19·5 | *x* + 4 |

Either:
Rule to get from upper number to lower number is add 4.
Rule to get from lower number to upper number is subtract 4.

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 4 | 8 | 2 × *y* ÷ 5 |
| 2·5 | 5 | 10 | 20 | *y* |

Either:
Rule to get from upper number to lower number is halve and multiply by 5.
Rule to get from lower number to upper number is double and divide by 5.

### Reflect

Same: both rules involve the digit 5.

Different: the first rule involves multiplying *a* by 5 and the second rule involves adding 5 to *a*.

## Lesson 2: Finding a rule (2)

➜ pages 67–69

**1.** a)

| Week | 1 | 2 | 3 | 5 | 10 | 11 |
|---|---|---|---|---|---|---|
| Total savings | 28 | 31 | 34 | 40 | 55 | 58 |

b) After *y* weeks, Olivia has saved 25 + 3 × *y* pounds.

**2.** Number line showing jumps of £4 backwards from £50.

| Week | 1 | 2 | 3 | 5 | 10 | *n* |
|---|---|---|---|---|---|---|
| Money left | 46 | 42 | 38 | 30 | 10 | 50 – 4*n* |

After *n* weeks, he has 50 – 4 × *n* pounds left.

**3.**

| Number of triangles | 1 | 2 | 3 | 4 | 5 | 10 | 100 |
|---|---|---|---|---|---|---|---|
| Number of sticks used | 3 | 5 | 7 | 9 | 11 | 21 | 201 |

To make 1 triangle, 3 sticks are used.
To make 2 triangles, 5 sticks are used.
To make 3 triangles, 7 sticks are used.
To make *n* triangle, 1 + 2 × *n* sticks are used.

**4.** For *g* houses, you need 5 + 5 × *g* sticks.
(Accept or equivalent expression; for example: (*g* + 1) × 5)

**5.** a) For *n* squares, you need 2*n* + 2 circles.
*n* = 100, so 2*n* = 200
2*n* + 2 = 202 circles

b) Answers will vary; for example:
Two circles drawn in each square: For *n* squares, you need 2*n* circles.

### Reflect

Answers will vary; for example:

Emma puts £100 in a bank account and takes £3 out every week to pay for a trip to the swimming pool. After *y* weeks how much money is left in the account?

## Lesson 3: Using a rule (1)

➜ pages 70–72

**1.** a) If Richard has *x* guinea pigs, Luis has *x* + 2 guinea pigs.

b) Bar model with six sections labelled *x*, 2, *x*, 2, *x*, 2 (can be in any order).

c) Ambika has 15 guinea pigs.

d)

| | Number of guinea pigs | | | | |
|---|---|---|---|---|---|
| Richard | 1 | 2 | 5 | 10 | 20 |
| Luis | 3 | 4 | 7 | 12 | 22 |
| Ambika | 9 | 12 | 21 | 36 | 66 |

**2.** a)

| Input | 1 | 2 | 3 | 5 | 10 |
|---|---|---|---|---|---|
| Output | 5 | 10 | 15 | 25 | 50 |

If the input is $a$, the output is $5 \times a$ (which can be written as $5a$).

b)

| Input | 1 | 2 | 3 | 5 | 10 |
|---|---|---|---|---|---|
| Output | 7 | 12 | 17 | 27 | 52 |

If the input is $b$, the output is $5b + 2$.

c) Outputs will vary as children choose own inputs, for example:

| Input | 1 | 2 | 3 | 5 | 10 |
|---|---|---|---|---|---|
| Output | 15 | 20 | 25 | 35 | 60 |

If the input is $c$, the output is $5(2 + c)$ or $10 + 5c$.

d) Outputs will vary as children choose own inputs; for example:

| Input | 1 | 2 | 3 | 5 | 10 |
|---|---|---|---|---|---|
| Output | 10 | 20 | 30 | 50 | 100 |

If the input is $d$, the output is $10d$.

**3.**

| Input | 1 | 2 | 5 | 100 | 1,000 | $a$ |
|---|---|---|---|---|---|---|
| Output for − 10 | $^-9$ | $^-8$ | $^-5$ | 90 | 990 | $a - 10$ |
| Output for +5 − 15 | $^-9$ | $^-8$ | $^-5$ | 90 | 990 | $a + 5 = 15$ $= a - 10$ |

Yes, Max is correct since $a + 5 - 15 = a - 10$.

**4.** a) and b) There are many possible pairs of operations; for example:
+ 10 × 5; × 10 × 10; × 2 + 80
Children should complete the table according to their functions; for example:
+ 10 × 5 gives:

| Input | 10 | 20 | 30 | 40 | $x$ |
|---|---|---|---|---|---|
| Output | 100 | 150 | 200 | 250 | $5(x + 10)$ or $5x + 50$ |

## Reflect

No, Emma is not correct.

When $x = 100$: $3x + 2 = 300 + 2 = 302$
When $x = 10$: $3x + 2 = 30 + 2 = 32$
$32 \times 10 = 320$ which is not 302, Emma's suggestion does not work.

Reasons will vary; for example: Using the rule on $x = 10$ gives $(3 \times 10) + 2$. When you then multiply this answer by 10, this gives $3 \times 100 + 20$. This is not the same as the required output of $3 \times 100 + 2$.

# Lesson 4: Using a rule (2)

➜ pages 73–75

**1.** a) The total value is $5n$ pence.

b)

| Number of coins | Reena's total value |
|---|---|
| 4 | 5p × 4 = 20p |
| 5 | 5p × 5 = 25p |
| 10 | 5p × 10 = 50p |
| 30 | 5p × 30 = 150p |
| 50 | 5p × 50 = 250p |

**2.** a) Hiring of the court costs $20n$ pence (for $n$ minutes).

b)

| Time in minutes | Cost |
|---|---|
| $n$ | 20p × $n$ = 20p$n$ |
| 10 | 20p × 10 = 200p (=£2) |
| 30 | 20p × 30 = 600p (=£6) |
| 60 | 20p × 60 = 1,200p (=£12) |
| 120 | 20p × 120 = 2,400p (=£24) |

**3.**

| | $x + 30$ | $30 - x$ | $30x$ |
|---|---|---|---|
| $x = 5$ | 35 | 25 | 150 |
| $x = 10$ | 40 | 20 | 300 |
| $x = 30$ | 60 | 0 | 900 |
| $x = 0$ | 30 | 30 | 0 |

**4.** No, the order of the operations matters.
If Aki adds 5 then multiplies by 10 he would get $(7 + 5) \times 10 = 12 \times 10 = 120$.
The correct answer is $(7 \times 10) + 5 = 70 + 5 = 75$.

**5.** If $y$ is an even number then $5y$ will be a multiple of 10 so $100 - 5y$ will be a multiple of 10.

**6.** When $y = 1$, $10y - y = 9$.
Other examples will vary, depending on the choice of $y$ but $10y - y$ will always be equal to $9y$.
Diagrams could include bar models split into 10 sections marked $y$ with one subtracted.

## Reflect

Answers will vary; for example:

$y = 1$: $4 + 2y = 6$
$y = 5$: $4 + 2y = 14$

Doubling any whole number gives an even number, so $2y$ is always even. 4 is even and when you add two even numbers together the answer will also be even. So, the rule $4 + 2y$ always generates even numbers.

# Lesson 5: Using a rule (3)

➜ pages 76–78

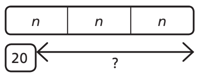
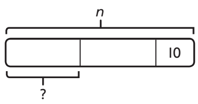
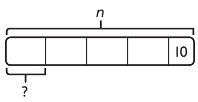
**1.** a) Length of ribbon left is $100 - 5y$.
b) There is 40 cm of ribbon left.

**2.** a) The total height is $15 + 10n$.
b) $15 + 10 \times 8 = 15 + 80$
The height is 80 cm.

**3.** a) A: $a + 50$, C: $\frac{a}{4}$ or $a \div 4$
B: $a - 50$ D: $50 + 3a$
b) A = 125 B = 25 C = 18·75 D = 275

**4.** Equivalent expressions matched:
5 less than $y$ ➜ $y - 5$
$y$ more than 20 ➜ $20 + y$
double $y$ ➜ $2y$

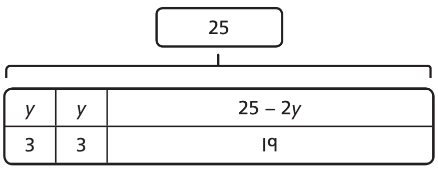
**5.**

| | Write an expression for each ?. | Substitute $n = 110$ into each expression. Calculate the value of ?. |
|---|---|---|
|  | $3n - 20$ | 310 |
|  | $\frac{n-10}{2}$ (or $(n - 10) \div 2$) | 50 |
|  | $\frac{n-10}{4}$ (or $(n - 10) \div 4$) | 25 |

## Reflect

When $y = 3$, $25 - 2y = 25 - 6 = 19$

Bar models may vary; for example:




# Lesson 6: Formulae

➜ pages 79–81

**1.** a) Formula: $3a$
Perimeter = 12 cm
b) Formula: $4a$
Perimeter = 16 cm
c) Formula: $2a + 2b$
Perimeter = 18 cm
d) Formula: $4a + 4b$
Perimeter = 36 cm

**2.** Tower A = 1,200 inches
Tower B = 2,400 inches
Tower C = 1,800 inches

**3.** $200 \times 48 = 9{,}600$
The rocket has travelled 9,600 miles.

**4.** Max is incorrect, since one side of each of the squares now lies inside the new shape. The perimeter of the new shape is $6a$; for example:
$a = 2$ cm, so perimeter of the new shape is
$6 \times 2 = 12$ cm.

**5.** Pattern A continued: $99 + 4 = 100 + 3$
$99 + 5 = 100 + 4$,
$99 + a = 100 + a - 1$
Described in words: Adding a number to 99 will always give the same answer as adding one less than the number to 100.
Pattern B continued: $99 \times 3 = 100 \times 3 - 3$,
$99 \times 4 = 100 \times 4 - 4$
$99 \times b = 100 \times b - b$
Described in words: Multiplying a number by 99 will always give the same answer as multiplying it by 100 and then subtracting one lot of the number.

## Reflect

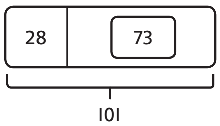
The formula for the perimeter is $2x + y$.

Substituting $x = 10$ and $y = 8$ into this expression gives $20 + 8 = 28$.

# Lesson 7: Solving equations (1)

➜ pages 82–84

**1.** a) Right-hand column completed: 250 350
Additional rows will vary depending on choice of $a$.
Check right-hand column = $a + 150$.
b) Right-hand column completed: 140 130 100
Additional rows will vary depending on choice of $b$.
Check right-hand column = $150 - b$.
c) $c = 101 - 28 = 73$

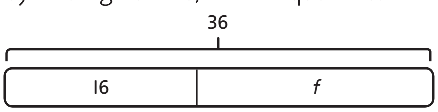



$c = 73$

**2.** a) Equation: $m + 50 = 500$; $m = 500 - 50 = 450$.
Mass of flour is 450 g.
b) Equation: $s - 25 = 250$; $s = 250 + 25 = 275$.
Bag originally contained 275 g of raisins.

**3.** a) $x - 10 = 300$
$x = 300 + 10 = 310$
b) $300 = 10y$
$y = 300 \div 10 = 30$
c) $z \div 10 = 300$
$z = 300 \times 10 = 3{,}000$

**4.** No, Luis is not correct. Explanations may vary; for example: The equation can be represented by a part-whole bar model where the whole is 36, one part is $f$ and the other part is 16. $f$ can therefore be worked out by finding $36 - 16$, which equals 20.



**5.** a) Equation: $10a = 2$
Solution: $a = 10 \div 2 = 0{\cdot}2$
b) Equation: $1{\cdot}5b = 150$
Solution $b = 150 \div 1{\cdot}5 = 100$
c) Equation: $c \div 10 = 2$
Solution: $c = 2 \times 10 = 20$
d) Equation: $d - 90{\cdot}9 = 909{\cdot}09$
Solution: $d = 909{\cdot}09 + 90{\cdot}9 = 999{\cdot}99$

### Reflect

Solution: $y = 125$

Methods will vary; for example:
Method 1: writing the equation as a bar model and using the inverse of +75 to subtract 75, i.e. $200 - 75 = 125 = y$.
Method 2 could involve substituting in different values of $y$ until finding that when $y = 125$, $y + 75 = 200$.

## Lesson 8: Solving equations (2)

➜ pages 85–87

**1.** a) $x + 25 = 40$
Subtract 25 from each scale.
$x = 15$
b) $3c = 150$
÷ each side by 3
$c = 50$
c) $a + 45 = 100$
$100 - 45 = 55$
$a = 55$
d) $5d = 150$
$150 \div 5 = 30$
$d = 30$

**2.** a) ➜ $c - 25 = 50$
$c = 75$
b) ➜ $25 = 5c$
$c = 5$
c) ➜ $25 + c = 50$
$c = 25$

**3.** a) $f = 3$  d) $i = 250$
b) $g = 2{\cdot}5$  e) $j = 36$
c) $h = 363$  f) $k = 1$

**4.** Answers will vary; for example:
$y + 8 = 10$  $80 \div y = 8$
$y = 2$  $y = 10$
$24 - y = 10$  $80 \times y = 240$
$y = 14$  $y = 3$

### Reflect

Answers will vary; for example:

Bar model where the whole is 100, one part is $x$ and the other part is 90.

Other diagrams could include balance scales with 100 on one side and 90 and $x$ on the other.

## Lesson 9: Solving equations (3)

➜ pages 88–90

**1.** a) $3a + 2 = 17$
$-2 \quad -2$
$3a = 15$
$\div 3 \quad \div 3$
$a = 5$
b) $4b + 80 = 100$
$b = 20$

**2.** $50 = 15 + 5c$
$35 = 5c$
$c = 7$

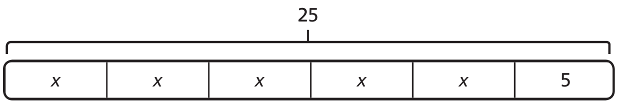
**3.** $3y + 5 = 80$
$3y = 75$
$y = 25$

**4.** $6n + 3 = 50 + 1$
$6n + 3 = 51$
$6n = 48$
$n = 8$

**5.** a) $a = 20$  c) $b = 14$
b) $c = 65$  d) $d = 15$

**6.** a) $(x \div 5) - 5 = 6$
$x \div 5 = 11$
$x = 55$
b) $(z + 20) \times 10 = 1{,}000$
$z + 20 = 100$
$z = 80$

### Reflect




## Lesson 10: Solving equations (4)

➜ pages 91–93

**1.** a)

| Perimeter | j = ? | k = ? |
|---|---|---|
| 12 cm | 1 cm | 5 cm |
| 12 cm | 2 cm | 4 cm |
| 12 cm | 3 cm | 3 cm |
| 12 cm | 4 cm | 2 cm |
| 12 cm | 5 cm | 1 cm |

b) The greatest area, of 9 $cm^2$, occurs when $j = 3$ cm and $k = 3$ cm.

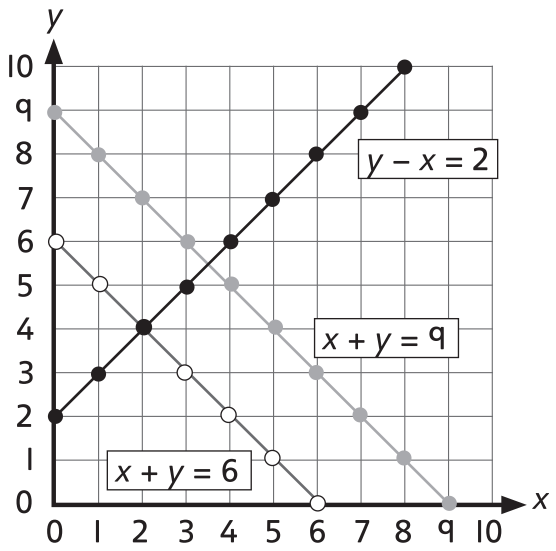
**2.** Equation: $a + b = 4$
Table completed showing pairs that total 4 kg.
Answers may vary; for example:

| *a* = ? | *b* = ? |
|---|---|
| 1 kg | 3 kg |
| 2 kg | 2 kg |
| 3 kg | 1 kg |
| $3\frac{1}{2}$ kg | $\frac{1}{2}$ kg |
| 0·6 kg | 3·4 kg |

**3.** Equation: $e \times f = 100$.
All possible solutions should be shown (may be in different order):

| *e* = ? | *f* = ? |
|---|---|
| 1 m | 100 m |
| 2 m | 50 m |
| 4 m | 25 m |
| 5 m | 20 m |
| 10 m | 10 m |
| 20 m | 5 m |
| 25 m | 4 m |
| 50 m | 2 m |
| 100 m | 1 m |

**4.**




**5.** a) The four numbers must be 1, 3, 5 and 11 or 1, 3, 7 and 9 (but be added in any order giving 24 calculations for each set).

b) There are 14 possible calculations:

| | |
|---|---|
| 1 + 2 − 1 | 5 + 4 − 7 |
| 3 + 2 − 3 | 7 + 4 − 9 |
| 5 + 2 − 5 | 1 + 6 − 5 |
| 7 + 2 − 7 | 3 + 6 − 7 |
| 9 + 2 − 9 | 5 + 6 − 9 |
| 1 + 4 − 3 | 1 + 8 − 7 |
| 3 + 4 − 5 | 3 + 8 − 9 |

## Reflect

Answers will vary; for example:

Drawing a table helps, particularly if you list possibilities methodically starting either at the lowest or highest, finishing when the numbers start to repeat.

# Lesson 11: Solving equations (5)

→ pages 94–96

**1.** Two possible solutions:
3 × 5p and 5 × 2p    1 × 5p and 10 × 2p
25p could also be made using 5 × 5p coins but this would not match the criteria since Alex also has 2p coins.

**2.** Assuming lengths are whole numbers, there are six possible solutions:
$a = 1$ cm, $b = 11$ cm (area = 11 cm$^2$)
$a = 11$ cm, $b = 1$ cm (area = 11 cm$^2$)
$a = 2$ cm, $b = 10$ cm (area = 20 cm$^2$)
$a = 10$ cm, $b = 2$ cm (area = 20 cm$^2$)
$a = 3$ cm, $b = 9$ cm (area = 27 cm$^2$)
$a = 9$ cm, $b = 3$ cm (area = 27 cm$^2$)

**3.** Equation: $4b + 8r = 32$
There are 5 possible solutions:
$b = 8, r = 0$    $b = 6, r = 1$    $b = 4, r = 2$
$b = 2, r = 3$    $b = 0, r = 4$

**4.** a) $50a - 25b = 100$. Solutions given will vary; for example:
$a = 2, b = 0$: 100 − 0 = 100
$a = 3, b = 2$: 150 − 50 = 100
$a = 4, b = 4$: 200 − 100 = 100
$a = 5, b = 6$: 250 − 150 = 100
$a = 10, b = 16$: 500 − 400 = 100
Pattern: For every 1 *a* goes up, *b* goes up 2.

b) $50 + c = d - 150$. Solutions given will vary; for example:
$c = 50, d = 250$: 50 + 50 = 250 − 150
$c = 100, d = 300$: 50 + 100 = 300 − 150
$c = 150, d = 350$: 50 + 150 = 250 − 150
$c = 0, d = 200$: 50 + 0 = 200 − 150
$c = 800, d = 1{,}000$: 50 + 800 = 1,000 − 150
Pattern: *c* is always 200 smaller than *d*.

**5.** The only numbers less than 20 which are the sum of two square numbers are: 5, 10, 13 or 17. It is not possible to make a total of 11 by adding two prime numbers. Therefore, the combinations of possible choices with a difference of 1 are:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Bella** | 4 (2 + 2) | 6 (3 + 3) | 9 (2 + 7) | 12 ( 5 + 7) | 14 (3 + 11) | 16 (5 + 11) | 18 (7 + 11) |
| **Danny** | 5 (1 + 4) | 5 (1 + 4) | 10 (1 + 9) | 13 (4 + 9) | 13 (4 + 9) | 17 (1 + 16) | 17 (1 + 16) |

## Reflect

Answers will vary; for example:

$6x + 2y = 28$

Solutions are $x = 1$, $y = 11$; $x = 2$, $y = 8$; $x = 3$, $y = 5$; $x = 4$, $y = 2$

# End of unit check

**➜ pages 97–98**

## My journal

**1** a) $3a + 5 = 20$
Answers will vary; for example:
Kate puts £5 in the bank, and saves a set amount each week. After 3 weeks she has £20. How much does she save each week?

b) $5b - 8 = 17$
Answers will vary; for example:
Kate saves a set amount each week. After 5 weeks she withdraws £8, leaving £17. How much does she save each week?

## Power puzzle

There are 15 different types of rectangles:

2 × 1 rectangles, 1 × 2 rectangles, 3 × 1 rectangles,
1 × 3 rectangles, 4 × 1 rectangles, 1 × 4 rectangles,
2 × 2 squares, 3 × 3 squares, 4 × 4 squares,
2 × 3 rectangles, 3 × 2 rectangles, 2 × 4 rectangles,
4 × 2 rectangles, 4 × 3 rectangles, 3 × 4 rectangles.