Unit 3: Four operations (2)

Lesson 1: Common factors

➜ pages 61–63

1. a) 1 × 14 = 14
2 × 7 = 14
1 × 18 = 18
2 × 9 = 18
3 × 6 = 18
The factors of 14 are 1, 2, 7 and 14.
The factors of 18 are 1, 2, 3, 6, 9 and 18.

b) The common factors of 14 and 18 are 1 and 2.

c) Children can draw diagrams to show that 14 does not form into an array with rows of 6. So 6 is not a factor of 14 and it therefore cannot be a common factor of 14 and 18.

2. Factors of 40: 1 × 40; 2 × 20; 4 × 10; 5 × 8

Factors of 100: 1 × 100; 2 × 50; 4 × 25; 5 × 20; 10 × 10

The common factors of 40 and 100 are: 1, 2, 4, 5, 10, 20

3. 8 is in the wrong place because it is a factor of both 80 and 200. 8 × 10 = 80; 8 × 25 = 200

5 is in the wrong place because it is a factor of both 80 and 200. 5 × 16 = 80; 5 × 40 = 200

4 a)

Factors of 35	Factors of 50	Factors of 70
1	1	1
5	2	2
7	5	5
35	10	7
	25	10
	50	14
		35
		70

b) Answers may vary but must be a multiple of 60. The lowest common factor of 1, 2, 3, 4 and 5 is 60, so any multiple of 60 will be a common factor.

Reflect

Common factors of 15 and 60: 1, 3, 5, 15

No, you would not need to check all the numbers up to 60. All the common factors must be factors of 15 so you would only need to check all the numbers up to 15.

Lesson 2: Common multiples

➜ pages 64–66

1.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The common multiples of 6 and 8 up to 100 are 24, 48, 72 and 96.

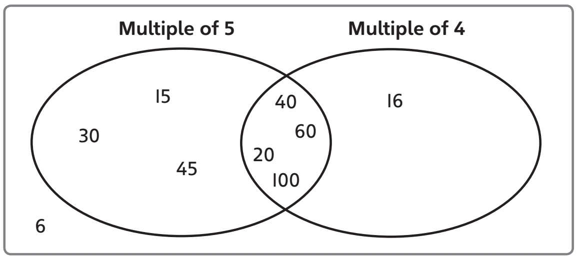
2. a)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

b)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

3.



Description may vary, for example: I notice that all the common multiples of 4 and 5 are multiples of 20.

4. 240, 300 and 360

5. a) The bar model shows that 48 is divisible by 12 exactly and it is also divisible by 4 exactly. Therefore 48 is a multiple of 12 and a multiple of 4, so it is a common multiple of 12 and 4.

b) No, the lowest common multiple of 4 and 12 is 12, so the common multiples up to 100 would be all multiples of 12 up to 100. Andy has missed out 12, 24, 36, 60, 72 and 84.

Reflect

Answers may vary but all must be multiples of 100.

Encourage children to find the lowest common multiple, which is 100. All other common multiples will be multiples of 100.

Lesson 3: Recognising prime numbers up to 100

→ pages 67–69

1. Children to show 7 by 7 array to demonstrate that 49 has a factor of 7.
49 ÷ 7 = 7.
So, factors of 49 are 1, 7 and 49.

2. I know 51 is not a prime number because it has factors 1, 3, 17 and 51. (Alternatively, children may just give a factor which is not 1 or 51, for example they may say that 3 is a factor of 51).

I know 55 is not a prime number because it has factors 1, 5, 11 and 55. (Alternatively, children may just give a factor which is not 1 or 55, for example they may say that 5 is a factor of 55.)

53 is a prime number because it only has two factors, 1 and itself (53).

3.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

4. Children should write two numbers in each cell from the following possible answers:
Top left cell: 2, 5
Bottom left cell: 1, 4, 10, 20, 25, 50, 100
Top right cell: Any prime number except 2 and 5
Bottom right cell: Any non-prime numbers except 1, 4, 10, 20, 25, 50 and 100
The top left section can have no more numbers in it as they are the only two factors of 100 that are also prime.

5. Explanations may vary, for example:
No, I do not agree. I know that 99 has a factor of 3, so if I partition 123 into 99 + 24, I know that 24 also has a factor of 3. Therefore 123 must have a factor of 3 so it is not prime.
This shows that if a number is prime, adding on 100 will not necessarily give a prime number.

Reflect

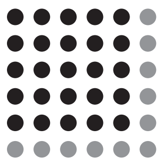
Explanations may vary. Encourage children to explain that they can work out prime or composite numbers using times-table and division knowledge or by drawing arrays. 85 is not prime as it is in the 5 times-table, so it has a factor of 5. 89 is prime – a multiplication tables grid shows that it is not a multiple of any number between 2 and 10 and so it only has two factors, 1 and itself.

Lesson 4: Squares and cubes

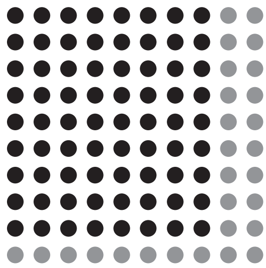
→ pages 70–72

1. a) 49 circled; 7 × 7 = 49
b) 125 circled

2. a)



b)



3. a) 81 d) 8 g) 1
b) 100 e) 4 h) 1
c) 121 f) 4 i) 2

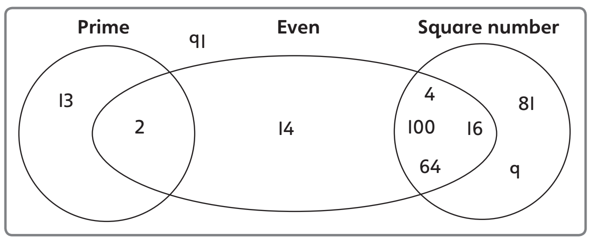
4. 72 more cubes need to be added. Explanations may vary, for example:

… because each layer is made from 6 × 6 cubes and you need 2 more layers to complete the big cube.
6 × 6 × 2 = 72.

… because there are 6 × 6 × 4 = 144 cubes in the shape whereas 6 × 6 × 6 = 216. 216 – 144 = 72.

5. Bella is incorrect as 30 × 30 = 900. She only multiplied 30 by 3 and not by 30.

6.



All square numbers can be written as $a \times a$, for some whole number a. Square numbers (apart from 1) therefore have more than two factors since their factors include 1, a and the number itself. The square number 1 is not prime as it has only one factor, 1 (itself). So, there are no prime square numbers and the circles do not need to overlap.

Reflect

Corrected equations: $1^2 = 1$; $3^2 = 9$; $5^3 = 125$

Comments may vary, for example:

Danny has worked out 1×2 but this is not the same as 1^2. Danny needs to remember that when you square a number you multiply it by itself so $1^2 = 1 \times 1 = 1$.

$9^2 = 9 \times 9 = 81$ so it is not true that $9^2 = 3$. Danny has squared the wrong number as it is true that $3 \times 3 = 9$ so $3^2 = 9$.

Danny has worked out 5×3 but this is not the same as 5^3. Danny needs to remember that when you cube a number you multiply it by itself and then by itself again so $5^3 = 5 \times 5 \times 5 = 125$.

Lesson 5: Order of operations

➜ pages 73–75

1. Lines drawn to match:
$3 \times 2 + 6$ → second image (towers of cubes)
$3 + 2 \times 6$ → third image (bead string)
$3 \times 6 + 2$ → first image (ten frames)

2. a) $5 + 1 \times 5 = 10$ Image should show 5 counters (1 group of 5) and another 5 counters.
b) $5 \times 2 - 5 = 5$ Image should show 5 groups of 2 counters (or 2 groups of 5 counters), with 5 counters crossed out.

3. a) $36 - 3 = 33$
b) $20 + 140 = 160$
c) $10 - 8 = 2$
d) $800 - 8 = 792$
e) $50 - 5 = 45$
f) $64 - 56 = 8$

4. a) 36; 180
b) 48; 320
c) 60; 5
d) 120; 5

5. a) 50
18
500
b) Answers will vary. Each calculation should have the same number in both boxes so that the answer to the division is 1.
Explanations will vary, for example:
Each pair of missing numbers involves the same number in each box.
The dividend and divisor are always the same number to give a quotient of 1.

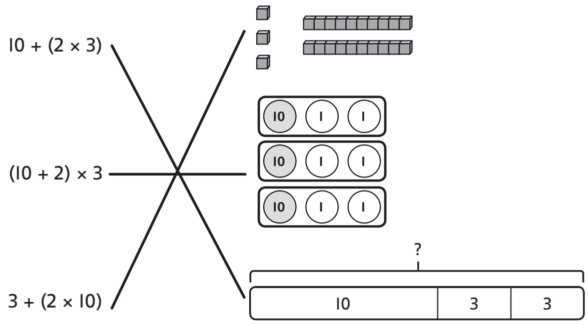
Reflect

Answers will vary – encourage children to write the multiplication and division part as the second operation in the calculation so that they cannot get it correct accidentally by just working from left to right.

Lesson 6: Brackets

➜ pages 76–78

1.



2. a) 100; $25 \times 4 = 100$
b) 9
c) 75
d) 3

3. a) Circled: $12 \times (3 + 5)$
b) $(3 + 5) \times 15 = 120$
c) $(5 \times 3) + (3 \times 5)$. This can also be written without brackets.

4. a) <
b) >
c) =

5. a) Answers may vary. Possible solutions include:
$(2 + 2 + 2) \times 2 = 12$; $2 \times (2 + 2 \times 2) = 12$
b) Answers may vary. Possible solutions include:
$10 = 3 \div 3 + 3 \times 3$; $10 = (3 \times 3) + (3 \div 3)$

6. a) Answers may vary. Possible solutions include:
Greater than 100: $(10 + 10) + (10 \times 10) = 120$;
$10 \times 10 + 10 \div 10 = 101$; $10 \times 10 \times (10 + 10) = 2{,}000$
Between 0 and 1: $(10 \div 10) \div (10 \times 10) = 0{\cdot}01$;
$(10 - 10) \times 10 \times 10 = 0$; $(10 + 10 - 10) \div 10 = 1$
Less than 0: $(10 - 10) - 10 \times 10 = {}^{-}100$;
$(10 \div 10) - (10 \times 10) = {}^{-}99$; $10 - 10 \times 10 \times 10 = {}^{-}990$
b) Answers will vary as children are asked to give the largest and smallest results they can find.
Largest: $10 \times 10 \times 10 \times 10 = 10{,}000$
Smallest: $10 - 10 \times 10 \times 10 = -990$

Reflect

Explanations may vary – encourage children to prove, by solving the calculations, that the left side is greater than the right side.

$10 \times (3 + 4) > 10 \times 4 + 3$
$70 > 43$

Lesson 7: Mental calculations (1)

→ pages 79–81

1. a) 57
b) 396
c) 35 × 9 = 315; 10 × 35 = 350

2. a) Kate receives 3p change.
b) Ebo spends £4·75 in total. He receives £15·25 change.

3. a) 200
b) 250
c) 300
d) 225

4. Explanations may vary, for example: Sofia rounded 98 to 100 and worked out 6 × 100 = 600. She added 2 cm to each length of wood, so she needs to subtract 6 × 2 from her answer. Sofia's mistake was that she subtracted 6 not 12. The correct answer is 588 cm or 5 m and 88 cm.

5. Explanations may vary – encourage children to use mental methods to work out that 9 × 49 = 9 × 50 – 9 = 441. Then use mental maths to solve 9 × 25 = 10 × 25 – 25 = 225. Use subtraction to work out 441 – 225 = 216 and use addition to work out 441 + 225 = 666.

Reflect

Answers may vary – encourage an explanation of using number sense. For example, if the numbers in a calculation are near multiples of 10 it may be efficient to use rounding then adjust the answer; if an addition or subtraction calculation does not involve exchange or only one simple exchange, it may be easy to do mentally; if numbers are close together when finding the difference, then a counting up mental strategy could be used.

Lesson 8: Mental calculations (2)

→ pages 82–84

1. a) 250 – 20 = 230
250,000 – 20,000 = 230,000
The remaining counters represents two hundred and thirty thousand.
b) 115 + 5 = 120
115,000 + 5,000 = 120,000
Now Ambika can represent 120,000.

2. a) 354,000
b) ninety-three thousand
c) three hundred thousand
d) 3,205,500

3. a) 49,000
b) 800,000
c) 850,000

4. a) 900
b) 9,000
c) 5
d) 19,000

5.

1,000 less	100 less	Number	100 more	1,000 more
99,001	99,901	100,001	100,101	101,001
999,001	999,901	1,000,001	1,000,101	1,001,001
899,500	900,400	900,500	900,600	901,500
8,101	9,001	9,101	9,201	10,101

6. a) 424,900
b) Solution can be any number between 1,800,010 and 2,000,010 (but not 1,800,010 or 2,000,010 themselves).

Reflect

Answers will vary – encourage an explanation that the calculations that can easily be solved mentally will involve limited exchange, for example, addition or subtraction of multiples of powers of 10. Calculations not easily solved mentally will involve multiple exchanges.

Lesson 9: Reasoning from known facts

→ pages 85–87

1. a) 5 × 6 × 7 = 210
b) 6 × 5 × 5 = 150
c) 3 × 7 × 9 = 189
d) 5 × 8 × 7 = 280

2. a) 425 + 85 = 510
b) 14 × 84 = 1,176
c) 4 × 164 = 656

3. Jamilla has multiplied by the difference between 148 and 48, instead of adding 6 lots of the difference.
To get the correct answer: 148 × 6 = (100 × 6) + (48 × 6).
As she already knows 48 × 6 = 288,
148 × 6 = 600 + 288 = 888.

4. a)

256 ÷ 16 = 16

16 × 17 = $\boxed{272}$

16 × 16 = $\boxed{256}$

2,560 ÷ 16 = $\boxed{160}$

$\boxed{25,600}$ = 160 × 160

$\boxed{256}$ = 8 × 32

32 × 16 = $\boxed{512}$

b) Answers will vary – ensure that children have used the related fact for their new equations, for example: 16 × 15 = 240; 2,560 ÷ 160 = 16; 16^2 = 256

5. a) $251 \times 11 = 2{,}761$
b) $65^2 = 4{,}225$
c) $25 \times 81 = 2{,}025$

Reflect

Answers may vary – encourage children to write facts that include doubling or multiplying by a power of ten, and/or using the inverse, for example: $85 \times 6 = 510$; $255 \div 3 = 85$; $85 \times 30 = 2{,}550$.

End of unit check

➜ pages 88–89

My journal

Olivia is correct as $30^2 = 30 \times 30 = 900$

Mo's idea is also correct, as $29 \times 30 = 30^2 - 30$.
So, $29 \times 30 = 900 - 30 = 870$

Power puzzle

Yes, you can make any whole number by adding 2 or 3 prime numbers.

Here are some possible solutions:

4 = 2 + 2	15 = 5 + 5 + 5
5 = 2 + 3	16 = 7 + 7 + 2
6 = 3 + 3	17 = 7 + 7 + 3
7 = 2 + 2 + 3	18 = 13 + 5
8 = 5 + 3	19 = 17 + 2
9 = 3 + 3 + 3	20 = 17 + 3
10 = 5 + 5	
11 = 5 + 3 + 3	100 = 97 + 3
12 = 5 + 5 + 2	101 = 97 + 2 + 2
13 = 5 + 5 + 3	
14 = 7 + 7	200 = 197 + 3