St. Katharine's Knockholt Church of England (Aided) Primary School

St Katharine's School Knockholt Calculation Policy

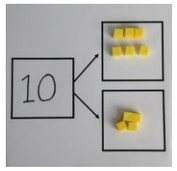
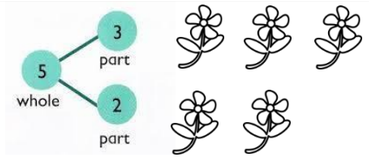
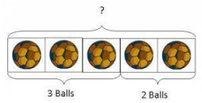
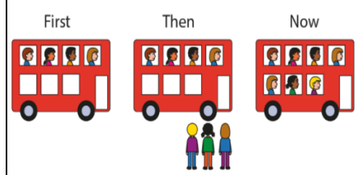
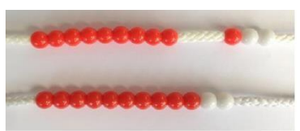
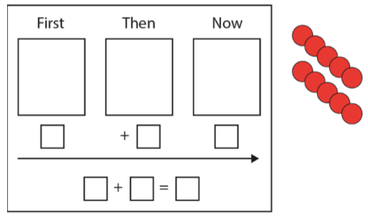
Maths at St Katharine's School Knockholt. We follow the guidelines of the National Curriculum and have developed teaching for mastery in maths across all classes. Teachers use Maths No Problem to help plan lessons in which children learn using small steps to build on previous understanding. The concrete-pictorial-approach is used by all teachers, in all year groups.

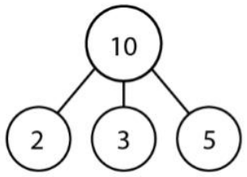
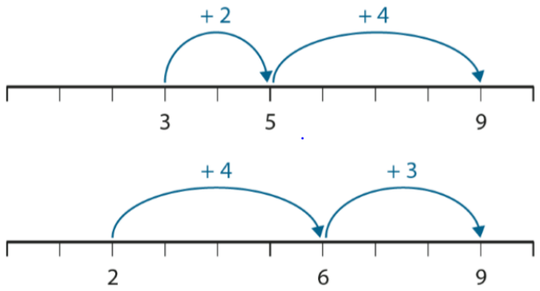
The Primary National Curriculum 2014 sets out the maths curriculum by year group. The curriculum aims to ensure that all pupils: become fluent in the fundamentals of mathematics, , so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately; are able to reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language; and can solve problems by applying their mathematics to a variety of routine and nonroutine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

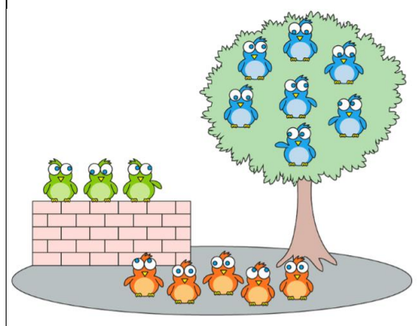
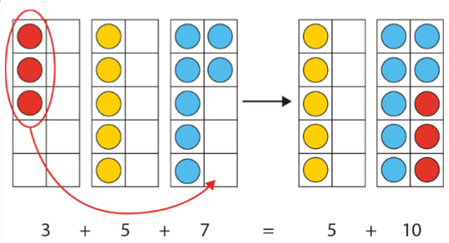
Teaching maths for mastery is a transformational approach to maths teaching which stems from high performing Asian nations such as Singapore. When taught to master maths, children develop their mathematical fluency without resorting to rote learning and are able to solve non-routine maths problems without having to memorise procedures. There is an inclusive approach where all children achieve and work in mixed ability groups. The pace of teaching is slower which results in greater progress.

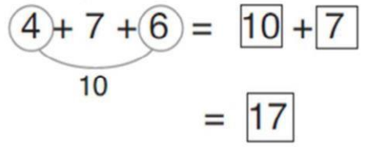
Concrete - pictorial – abstract (CPA). Children and adults can find maths difficult because it is abstract. The CPA approach builds on children's existing knowledge by introducing abstract concepts in a concrete and tangible way. It involves moving from concrete materials, to pictorial representations, to abstract symbols and problems. Concrete is the "doing" stage. During this stage, students use concrete objects to model problems. This may for be the real object or a counter/cube to represent the object. Pictorial is the "seeing" stage. Here, visual representations of concrete objects are used to model problems. This stage encourages children to make a mental connection between the physical object they just handled and the abstract pictures, diagrams or models that represent the objects from the problem. Abstract is the "symbolic" stage, where children use abstract symbols to model problems. The abstract stage involves the teacher introducing abstract concepts (for example, mathematical symbols). Children are introduced to the concept at a symbolic level, using only numbers, notation, and mathematical symbols (for example, +, –, x, /) to indicate addition, multiplication or division.

Addition

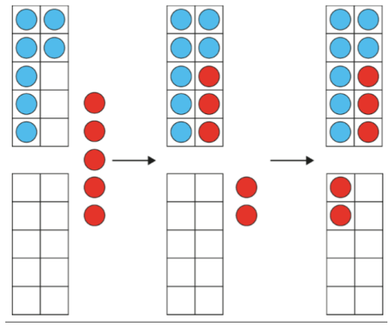
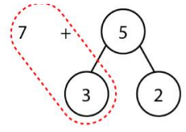
Steps of learning	Concrete	Pictorial	Abstract
Aggregation	There are four purple blocks and three orange blocks. There are seven blocks.    	 Use pictures to add two numbers together as a whole or in a bar.   	5 + 3 = 8 5 and 3 are the addends and 8 is the sum 3 + 5 = 8 3 are 5 the addends and 8 is the sum **Stem sentence:** **…….. and …….. are the addends and ……. Is the sum.** **Conceptual understanding: if we change the order of the addends the sum remains the same (commutative law)**
Augmentation	Act out stories with real children. First there were 4 children on the bus, then 3 got on, now there are 7 children on the bus.  First there were 9 beads, then 3 were added, now there are 12 	 Use counters as abstract representations of the story	☐ + 2 = 5 2 + ☐ = 7 Missing augend Missing addend **Stem sentence: first ……., then……, now…….** **Conceptual understanding: if I know two parts of the story I can work out the third part**

<table>
<tr>
<td>Add three single digits</td>
<td> 
3 blue marbles, 5 yellow marbles and 2 red marbles, altogether that makes 10</td>
<td> 
The 2 represents the red marbles, the 3 represents the blue marbles, the 5 represents the yellow marbles. There are 10 marbles altogether.</td>
<td> 
3 + 2 + 4 = 2 + 4 + 3

2 + 3 + 5 = 3 + 2 + 5
Conceptual understanding: when we add 3 numbers the total will be the same whichever pair we start with.</td>
</tr>
<tr>
<td>Make 10</td>
<td> 
If the total is more than 10 look to make 10 first</td>
<td> 
3 + 5 + 7 = 5 + 10

3 and 7 make 10 then we add on the 5</td>
<td> 
Combine the two numbers that make 10 and then add on the remainder.

Conceptual understanding: look for pairs/groups of three addends that sum to 10 first
8 + 3 + 6 + 1 = 10 + 6 = 16
Stem sentence: ……. + ……. + ……. makes 10 then 10 + ……. makes</td>
</tr>
<tr>
<td>Bridging through 10</td>
<td>Act out with real children:

There are 10 seats in a ride. 7 children have sat down. The carriage must be filled before starting a new one. 5 children arrive for the ride.</td>
<td> 
Show the same story with counters to represent children</td>
<td>First I partition the 5 into 3 and 2.
 
Then I add 7 and 3 to make 10. Then 10 and 2 make 12

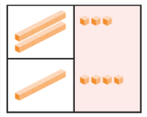
7 + 3 = 10 10 + 2 = 12

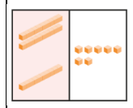
Stem sentence: first I partition the into …. + ……..,

then I add …to make 10, then 10 + …..</td>
</tr>
</table>

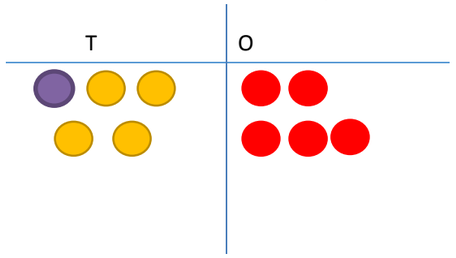
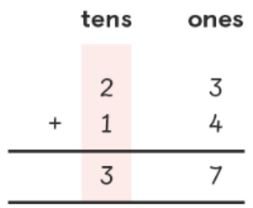
<table>
<tr>
<td>Column method - no regrouping</td>
<td>Make the two addends out of base ten.

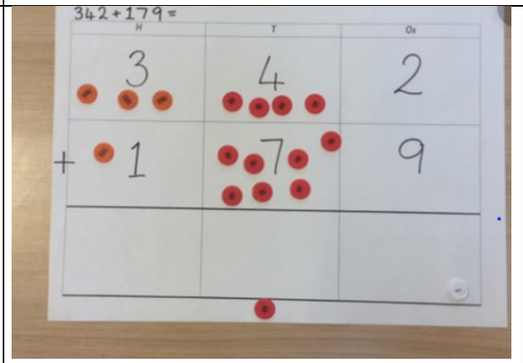
First add the ones.

 
4 + 3 = 7

Then add the tens. 20 + 30 = 50

23 + 14 = 57</td>
<td>After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.
 
</td>
<td> 

Formal written method</td>
</tr>
<tr>
<td>Column method – with regrouping</td>
<td> 
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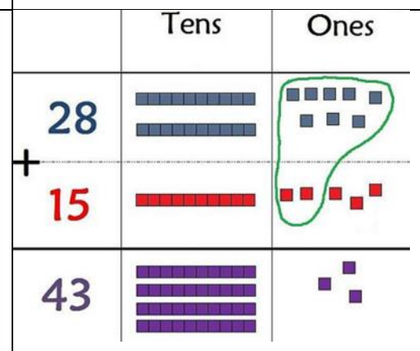
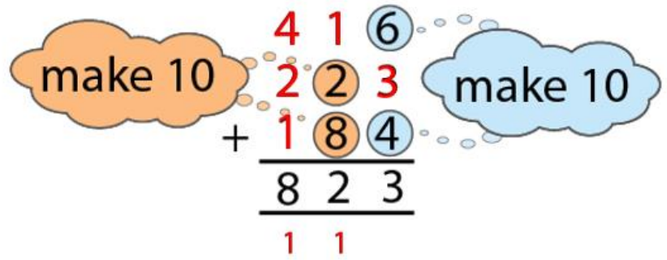
<td> 
</td>
<td> 
</td>
</tr>
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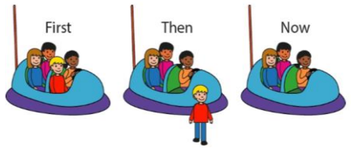
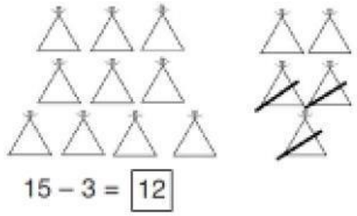
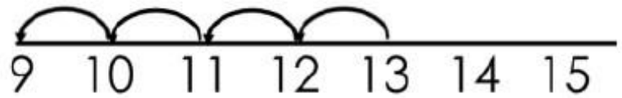
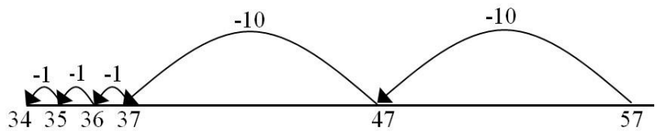
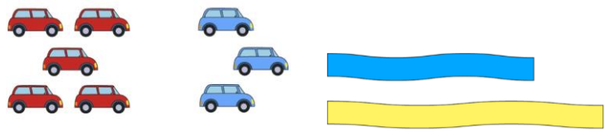
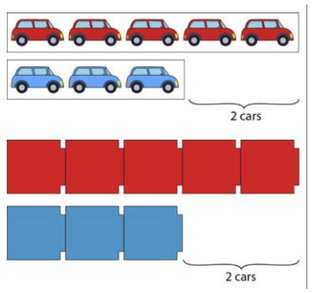
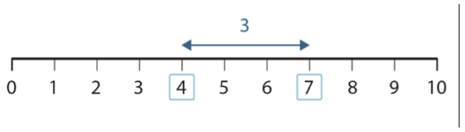
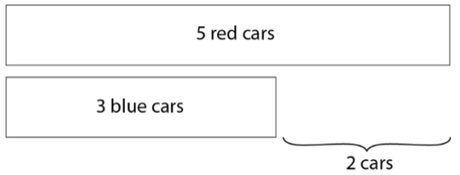
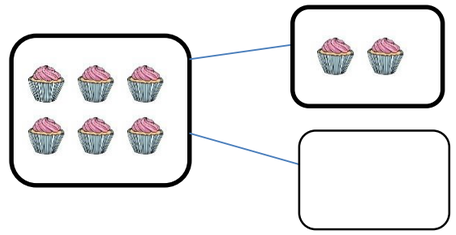
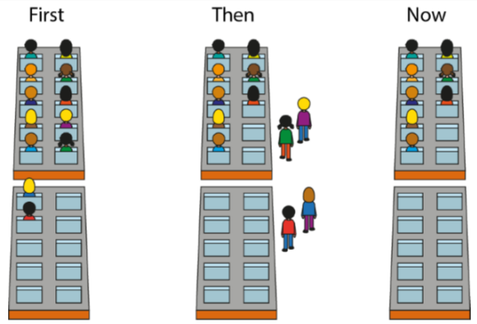
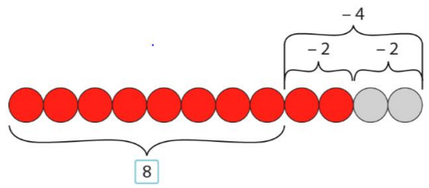
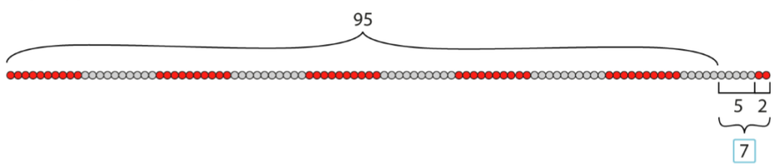
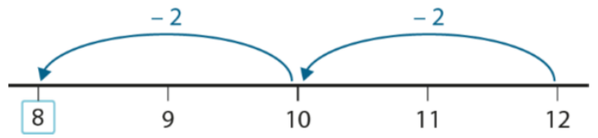
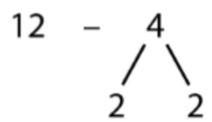
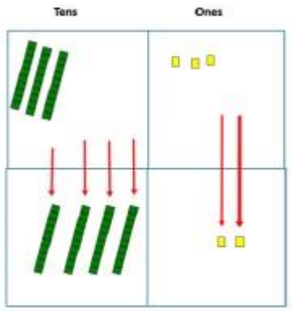
Subtraction

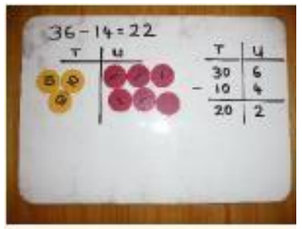
Learning steps	Concrete	Pictorial	Abstract
Subtraction as Reduction	Act out stories with real children.  First there were 4 children in the car, then 1 got out of the car, now there are 3 in the car	Cross out drawn objects to show what has been taken away.   	**18 – 3 = 15** **8 – 2 = 6**
Counting back strategy	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. **13 – 4**  Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers.	12 11 10 9  Put 13 in your head, count back 4. What number are you at? Use your fingers to help: put one finger up each time you count 1 less. Stop when 4 fingers are up.

Subtraction as difference	 Discrete items More red cars than blue cars Fewer blue cars than red cars Continuous measure Yellow is longer, blue is shorter	 Organise to show the difference Use picture/models to represent the items	 Difference is the gap: the difference between 4 and 7 is 3, the difference between 7 and 4 is 3.  Bar model to represent the relationship between numbers. Count up to find the difference.
Subtraction as partitioning	Act out stories with real children. 5 children are at the playground. 3 are playing on the swings. The rest are on the roundabout. How many are on the roundabout?  Use counters to represent the children	Use a pictorial representation to show the part-part whole model. 	**6 – 2 = 4** **Stem sentence: if …. is the whole and ……. is a part then …….is the other part**
Subtraction through 10	 First there were 12 children on the ride, then 4 got off, now there are 8. We first subtract the 2 children to get to 10, then we subtract 2 more children leaving 8 on the ride.	 Represent the children on the bead bar. 102 – 95 = 7 	Show it on a number line.   Show how you partition the 4 **12 – 2 = 10 10 – 2 = 8** **12 – 2 – 2 = 8**

Column method without exchange

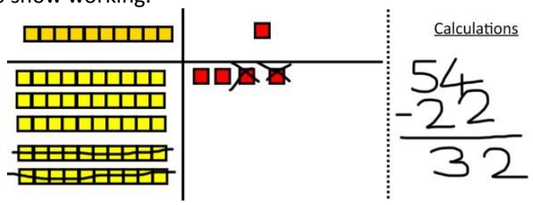


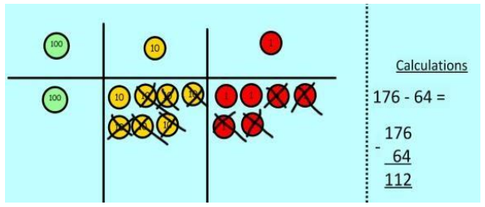
Use Base 10 to make the bigger number then take the smaller number away.



Show how you partition numbers to subtract. Again make the larger number first.

Draw the Base 10 or place value counters alongside the written calculation to help to show working.





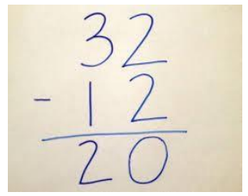
$47 - 24 = 23$

$40 + 7$

$-\ 20 + 4$

$20 + 3$

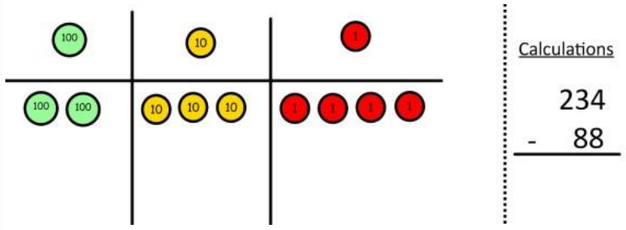
This will lead to a clear written column subtraction.



Column method with exchange

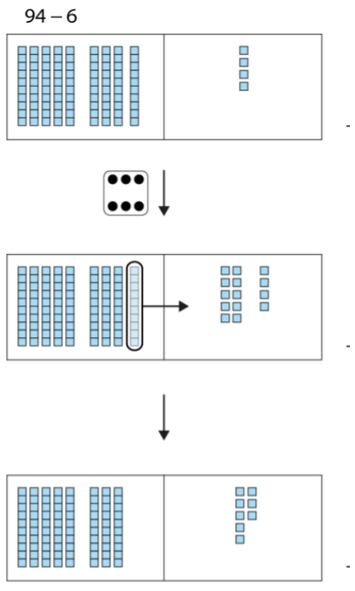
Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

Make the larger number with the place value counters



Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.

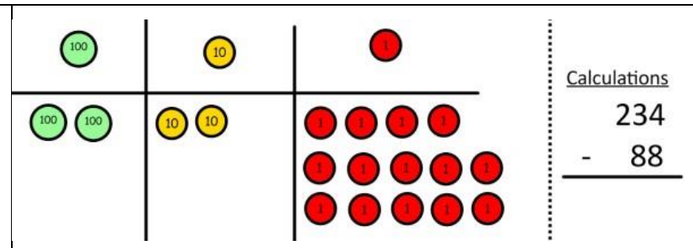
Draw the Dienes onto a place value grid and show the exchange of 1 ten to 10 ones the.



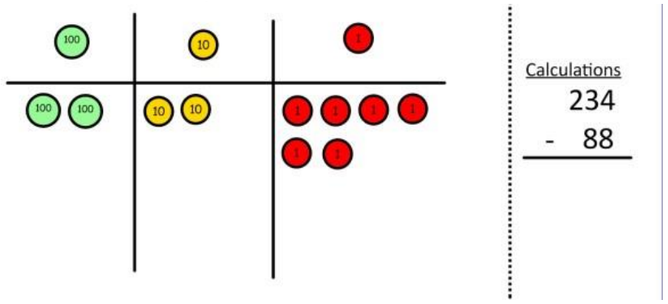
Show the algorithm for subtraction with exchange

10s	1s
~~9~~ 8	1 4
	6

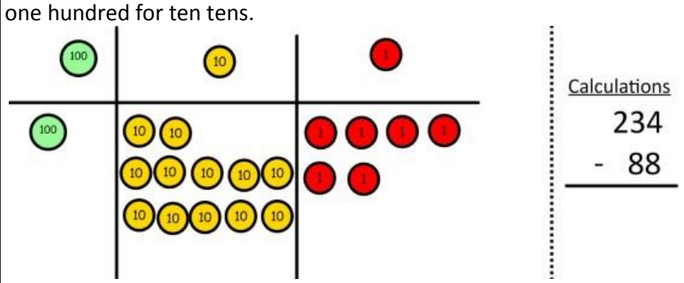
10s	1s
~~9~~ 8	1 4
	6
8	8



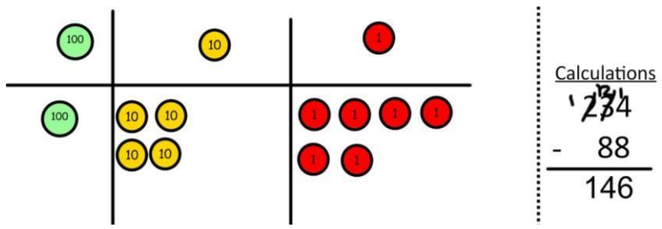
Now I can subtract my ones.



Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.

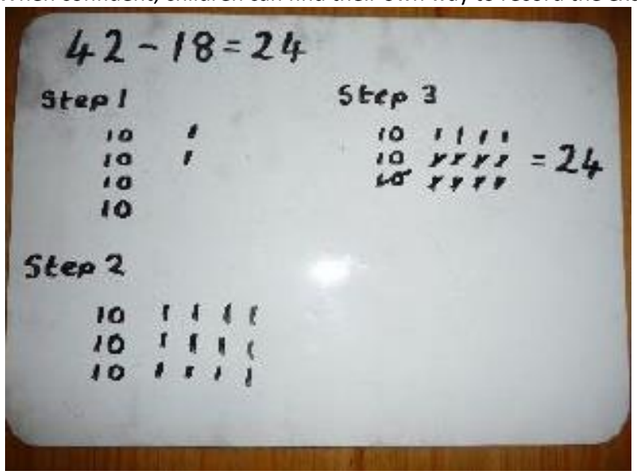


Now I can take away eight tens and complete my subtraction

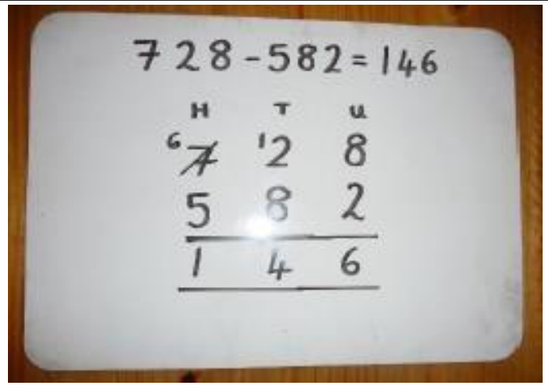


Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

When confident, children can find their own way to record the exchange



Just writing the numbers as shown here shows that the child understands the method and knows when to exchange.

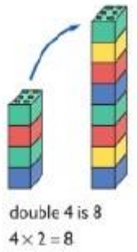
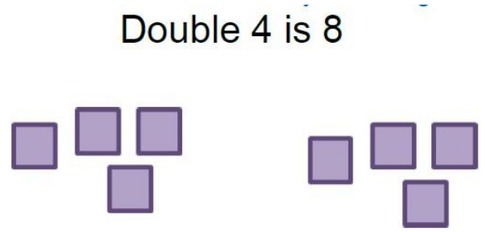
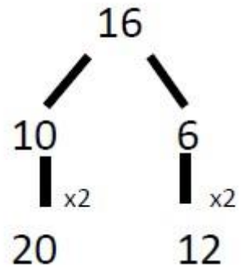
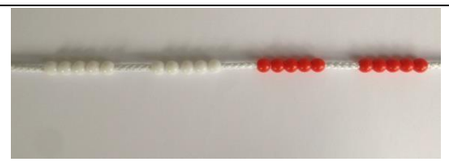
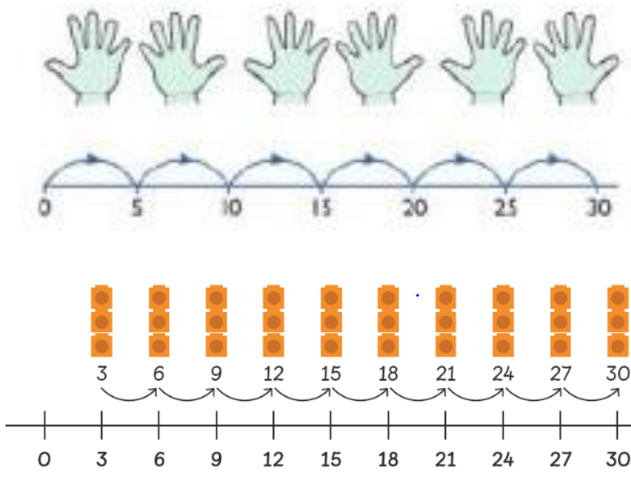
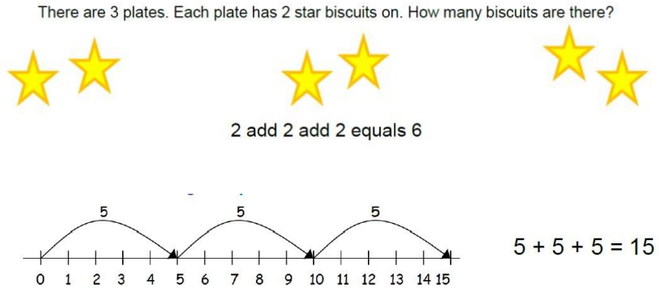


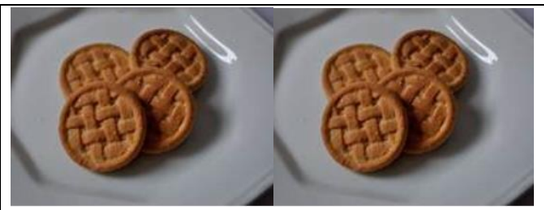
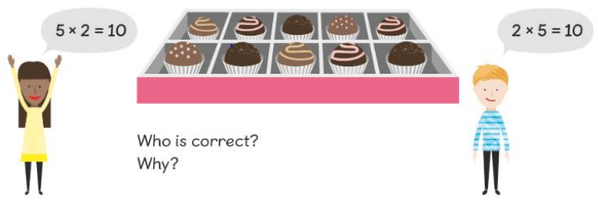
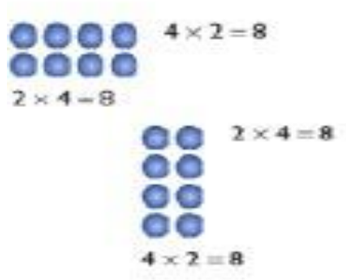
Moving forward the children use a more compact method.

This will lead to an understanding of subtracting any number including decimals.

$$\begin{array}{r} \ 5\ \ 12\ \ \ \ 1 \\ 2\ \ \cancel{6}\ \ \cancel{3}\ .\ \ 0 \\ -\ \ \ 2\ \ 6\ .\ \ 5 \\ \hline 2\ \ 3\ \ 6\ .\ \ 5 \end{array}$$

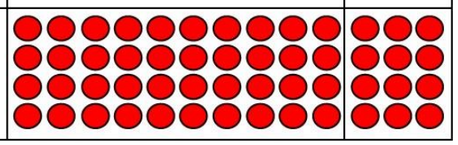
Multiplication

Learning steps	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.  	Draw pictures to show how to double a number.  	Partition a number and then double each part before recombining it back together. 
Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. **2, 4, 6, 8, 10** **5, 10, 15, 20, 25, 30**
Repeated addition	  Use different objects to add equal groups.	 	Write addition sentences to describe objects and pictures. 

One to many correspondence	 1 plate of 4 cookies 1 plate of 4 cookies 2 plates each with 4 cookies = 8 cookies	 Four groups of three	**1 x 3 = 3** **2 x 3 = 6** **3 x 3 = 9** **4 x 3 = 12**
Multiplication is commutative		 Draw arrays in different rotations to find **commutative** multiplication sentences.	Use an array to write multiplication sentences and reinforce repeated addition.

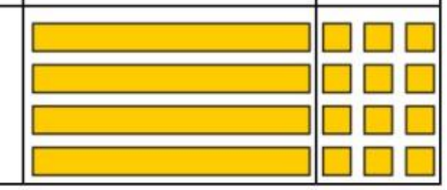
Multiplication follows law of distribution

Show the link with arrays to first introduce the grid method.

x	10	3
4		

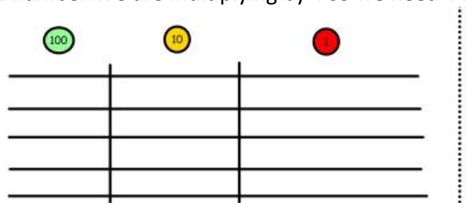
4 x 13 is shown as
4 rows of 10 and
4 rows of 3

Move on to using Base 10 to move towards a more compact method.

x	T	U
		

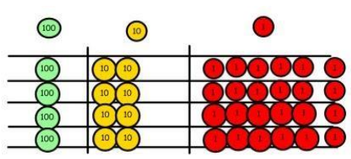
4 rows of 13

Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.



Calculations
4 x 126

Fill each row with 126.



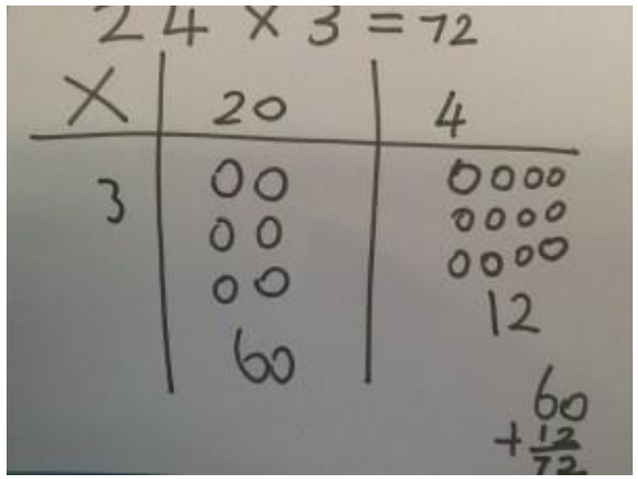
Calculations
4 x 126

400 + 80 + 24

Add up each column, starting with the ones making any exchanges needed.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

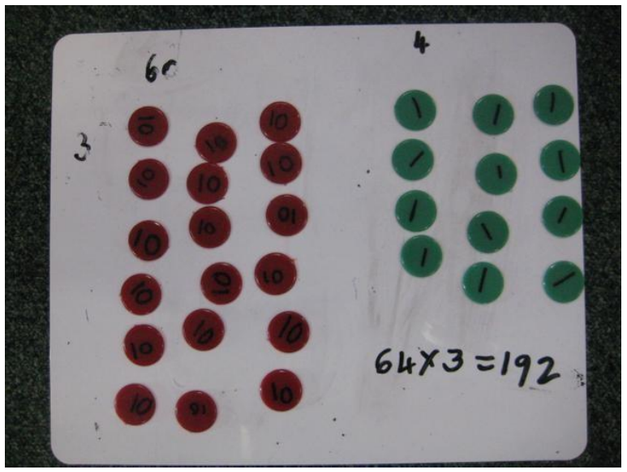
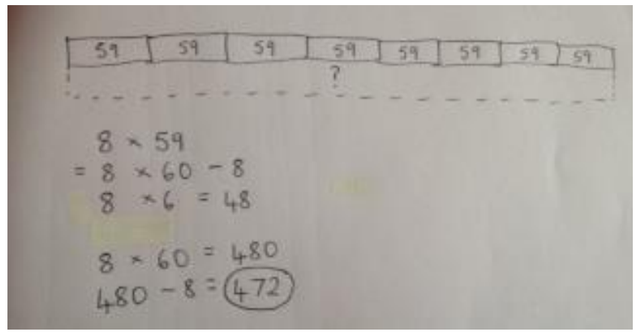
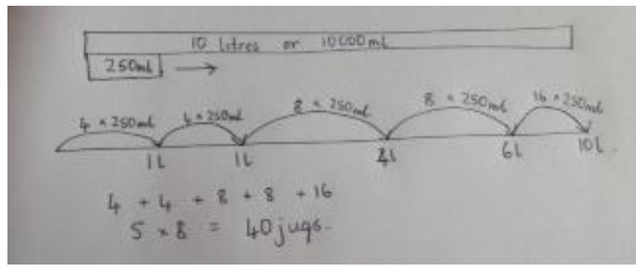
×	30	5
7	210	35

210 + 35 = 245

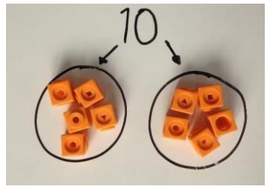
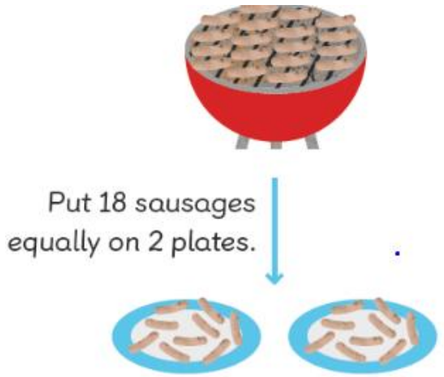
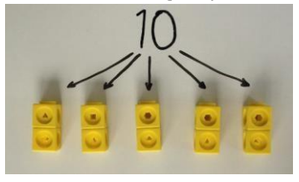
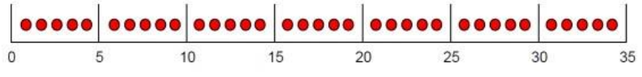
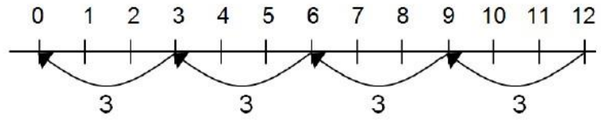
Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

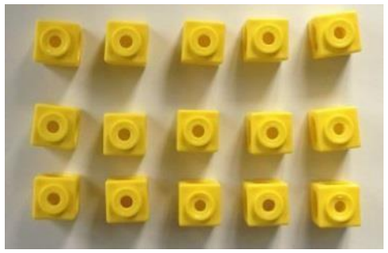
	10	8
10	100	80
3	30	24

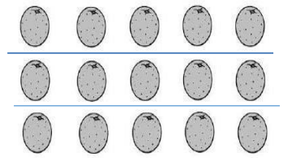
X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

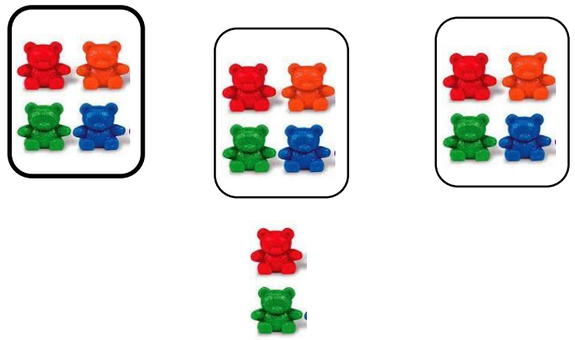
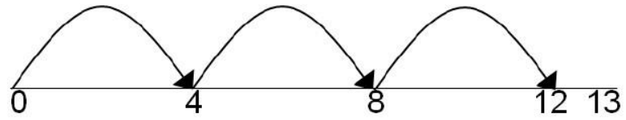
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication.  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.   	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer. 32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768 7 4 x 6 3 1 2 2 1 0 2 4 0 + 4 2 0 0 4 6 6 2 This moves to the more compact method. 2 3 1 1342 x 18 13420 10736 24156 1

Division

Objective and Strategies	Concrete	Pictorial	Abstract
Division as sharing	   I have 10 cubes, can you share them equally between 2 people?	Children use pictures to share quantities.  18 ÷ 2 = 9	**Share 9 buns between three people.** **9 ÷ 3 = 3**
Division as grouping	Divide quantities into groups of a given number Use cubes, counters, objects or place value counters to aid understanding. 10 divided into groups of2  35 ÷ 5 There are 7 groups of 5 in 35   12 ÷ 4 There are 3 groups of 4 in 12	Use a number line to show jumps in groups. The number of jumps equals the number of groups. 	**28 ÷ 7 = 4** **Divide 28 into groups of 7. How many groups of 7 are in 28?**

<table>
<tr>
<td>Division within arrays</td>
<td> 
Link division to multiplication by creating an array and thinking about the number sentences that can be created.

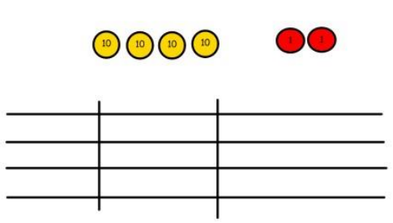
E.g. 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15</td>
<td>Draw an array and use lines to split the array into groups to make multiplication and division sentences.
3 groups of 5
  </td>
<td>Find the inverse of multiplication and division sentences by creating four linking number sentences.

7 x 4 = 28
4 x 7 = 28
28 ÷ 7 = 4
28 ÷ 4 = 7</td>
</tr>
<tr>
<td>Division with a remainder</td>
<td>14 ÷ 3 =
Divide objects between groups and see how much is left over
  </td>
<td>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.
 

Draw dots and group them to divide an amount and clearly show a remainder.
 
</td>
<td>Complete written divisions and show the remainder using r.
 
</td>
</tr>
</table>

Division of two-digit numbers

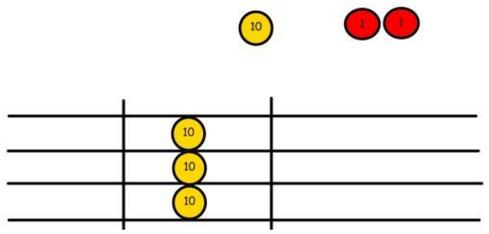
Use place value counters to divide using the bus stop method alongside



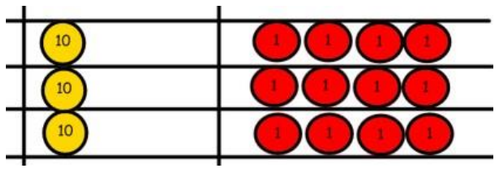
Calculations

42 ÷ 3

Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.

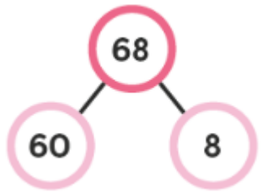


We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much in 1 group so the answer is 14.

Partition to make division easier



6 tens ÷ 2 8 ones ÷ 2

68 ÷ 2 = 30+ 4 = 34

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 218 \\ 4\overline{)872} \end{array}$$

Move onto divisions with a remainder.

$$\begin{array}{r} 86\ r\ 2 \\ 5\overline{)432} \end{array}$$

Finally move into decimal places to divide the total accurately.

$$\begin{array}{r} 14.6 \\ 35\overline{)511.0} \end{array}$$